

# Focal Properties of the Two-Tube Electrostatic Lens for Large Voltage Ratios

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Previous calculations of electron trajectories and first-order focal properties of the two-tube electrostatic lens have been extended to a voltage ratio of 1000. Considerable ultrafocal refraction occurs in these strong lenses, with the result that near the highest voltage ratio studied the two focal points are both on the low-voltage side of the lens and nearly coincident. The results are presented in the form of a table and  $P$ - $Q$  (image-object) curves.

## INTRODUCTION

In previous work,<sup>1-3</sup> accurate first-order focal properties for the two-tube electrostatic lens were calculated for voltage ratios from 1.5 to 50. Since then it has become evident that data for higher voltage ratios is needed. We therefore have extended these calculations to a voltage ratio of 1000.

## CALCULATIONS

Figure 1 shows the definition of the first-order focal properties of an electrostatic lens.<sup>4</sup> Rays A and B, parallel to the axis, enter the lens from left and right at distances  $T$  and  $T'$  from the axis, respectively, traverse the lens, and leave the lens field. The asymptotic rays are then extrapolated back to the lens, and their intersections with the axis and with the original rays define the apparent focal points  $F_1^*$ ,  $F_2^*$ , and principal planes  $H_1^*$ ,  $H_2^*$ , respectively, together with the apparent focal lengths  $f_1^*$ ,  $f_2^*$ .

Trajectories of incident parallel rays were computed for various values of  $T$  and  $T'$ , and apparent focal points and principal plane positions were calculated.

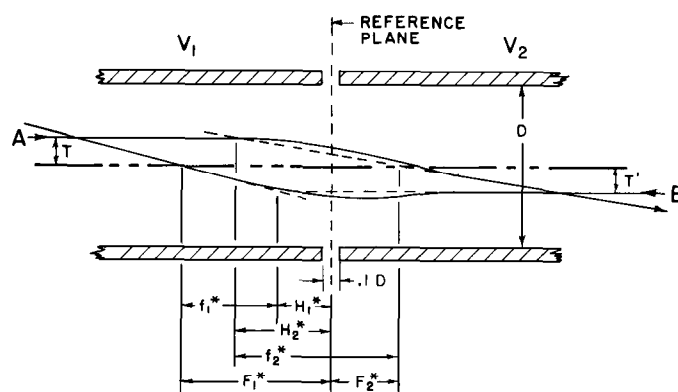


FIG. 1. Schematic drawing of the two-tube electrostatic lens, showing the definition of the apparent focal points  $F_1^*$  and  $F_2^*$ , principal planes  $H_1^*$  and  $H_2^*$ , and focal lengths  $f_1^*$  and  $f_2^*$ .

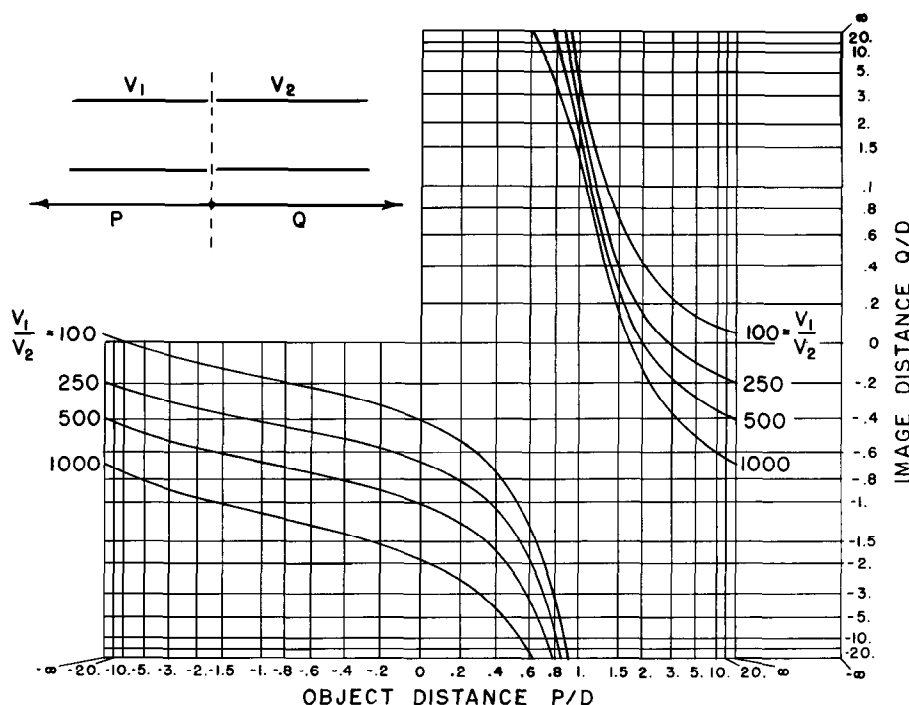


FIG. 2.  $P$ - $Q$  curves for the two-tube electrostatic lens.  $P$  is the object distance measured from the reference plane (see Fig. 1) and is positive to the left.  $Q$  is the image distance measured from the reference plane and is positive to the right.

The true focal points and principal plane positions were then taken to be the limit of the apparent values as the parallel rays approached the axis ( $T, T' \rightarrow 0$ ).

## RESULTS

The values obtained for the true focal lengths  $f_1$  and  $f_2$ , principal plane positions  $H_1$  and  $H_2$ , and positions of the focal points  $F_1$  and  $F_2$  are given in Table I. Negative values for  $F_2$  indicate that this focal point is on the left side of the lens, showing that considerable ultrafocal refraction occurs for these strong lenses. From a test of the relation  $f_1/f_2 = (V_1/V_2)^{1/2}$ , and a check of the expected quadratic dependence<sup>5</sup> of the apparent focal points and principal plane positions as a function of  $T, T'$ , we estimate that the accuracy of our first-order focal properties is  $\pm 0.1\%$ . We have not found any published data for comparison.

In Fig. 2 the image distance  $Q/D$  is plotted as a function of the object distance  $P/D$ , following Spangenberg and Field,<sup>6</sup> where  $D$  is the diameter of the lens. The relevant equation is

$$(P - F_1)(Q - F_2) = f_1 f_2. \quad (1)$$

In order to present data over a wide range of  $P$  and  $Q$  while giving linearity and continuity near  $P=0$  and  $Q=0$ , an arc tangent scaling of  $P$  and  $Q$  was used. Since it is common to have virtual objects and images in systems of several lenses, we have calculated  $P-Q$

TABLE I. First-order focal properties of the two-tube electrostatic lens.

$V_2/V_1$	$f_1/D$	$f_2/D$	$H_1/D$	$H_2/D$	$F_1/D$	$F_2/D$
100	0.2016	2.016	0.7155	1.9752	0.9171	0.04110
250	0.1590	2.515	0.6940	2.7121	0.8531	-0.1975
500	0.1478	3.305	0.6337	3.7030	0.7815	-0.3984
1000	0.1541	4.874	0.4747	5.5483	0.6288	-0.6741

curves for both real and virtual objects and images. In using these  $P-Q$  curves, any object or image inside the lens field (within about  $2D$  of the lens center) should be considered as virtual.

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